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MODELS FOR ALLOCATING STUDENTS TO FREE ELECTIVE COURSES

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The article is devoted to the development of mathematical models for allocating students to free elective courses in higher education institutions under conditions of limited capacity. The relevance of the study is determined by the need for transparent and formalized allocation mechanisms. Unlike early registration procedures, these mechanisms account for not only the order of application submission but also students' individual preferences and their priority based on an integral rating score. The aim of the study is to develop and provide a theoretical justification for mathematical models for allocating students to free elective courses. It also aims to identify the specific features of applying weighted and lexicographic approaches to such allocation. The study employs methods of discrete and lexicographic optimization. The allocation problem is formalized with due regard to course capacity constraints, the number of courses selected by each student, and feasibility conditions. Four mathematical models are constructed, combining ranked lists and normalized weight coefficients with two approaches to incorporating student priority, namely weighted and lexicographic. The experimental validation is carried out using generated data for 1000 students and 30 courses, using Monte Carlo simulation. It is established that the weighted models provide a more balanced allocation and a higher level of student satisfaction, whereas the lexicographic models ensure stricter adherence to the hierarchy of priorities. The scientific novelty lies in the development of a set of models that allow accounting not only the order but also the intensity of students' individual preferences. The practical significance of the results lies in the possibility of using the proposed models as a basis for creating transparent decision-support mechanisms in higher education institutions.

Keywords: free elective courses, student allocation, integer programming, lexicographic optimization, integral rating score.

1. Introduction

The problem of allocating students to elective courses lies at the intersection of higher education management, educational system design, and discrete optimization, and it has both scientific and practical significance. In its general form, the problem consists in the optimal allocation of limited educational resources (available places in elective courses) among students who have diverse preferences and competing priority levels.

Student-centered learning is a modern approach to organizing the educational process in which the main focus is placed on the student rather than on the teacher or the instructional material. This approach encourages students to participate in the educational process as autonomous and responsible actors. It also orients learning toward their needs and interests and supports the formation of individual educational trajectories. Increased responsibility, independence, and a thoughtful approach to learning are key features of student-centered learning.

The formation of an individual educational trajectory plays an important role in training qualified specialists. Responsibility for choosing academic disciplines also contributes to the development of a conscious and independent personality. An individual study plan of a higher education student consists of mandatory and elective academic disciplines chosen by the student.

The number of places in elective courses is usually limited due to insufficient staff resources, classroom capacity, the complexity of timetable formation, and high demand for certain courses. As a result, not all students can be enrolled in the educational components they have chosen. Therefore, there arises a need to apply certain criteria for selecting students when enrolling them in free elective courses. Such criteria may include:

1. academic performance;
2. year seniority;
3. relevance of the course to the student's profile;
4. inclusiveness and social factors;
5. registration time;
6. previous choice;
7. activity and motivation.

Academic performance. When choosing courses, students with a higher average grade for the previous period of study are given higher priority. This approach can stimulate conscientious learning and, importantly, is a transparent ranking criterion.

Year seniority. Senior students are given higher priority because they have less time left before graduation to choose courses.

Relevance of the course to the student's profile. Students for whom the chosen course corresponds to their specialty or educational program are given higher priority. For example, students of a certain faculty or specialty may have priority when choosing courses corresponding to their field of training.

Inclusiveness and social factors. Students from privileged categories, including persons with disabilities or orphans, may receive additional priority in the allocation process.

Registration time. Priority is given to students who first submitted an application for a particular course (FIFO). This approach motivates students to make their course selections on time and contributes to greater organization in the process of forming individual study plans.

Previous choice. Students who have previously studied a course similar in content may receive lower priority so that other students also have an opportunity to enroll in that course.

Activity and motivation. A student's participation in additional activities related to the course, in projects of a corresponding profile, and personal motivation stated in a special questionnaire may also be taken into account.

For example, at Igor Sikorsky Kyiv Polytechnic Institute, bachelor's students choose courses for the next academic year in the spring semester, having access to the same list of courses. All courses are divided into catalogs, within which a student may choose one or several courses depending on the year of study. Each course has a limited number of places. Students determine the priority of courses within the corresponding catalog. The current automated course allocation system at Igor Sikorsky Kyiv Polytechnic Institute relies on early registration. Preference is given to students who submitted their applications earlier. The algorithm allocates students to courses according to the order of submitted applications. If places are unavailable for the highest priority, the next priority is considered. Such an approach does not take into account academic achievements, individual characteristics of students, and other justified selection criteria.

Therefore, there is a need to develop formal mathematical models for allocating students to elective courses. These models should consider limited course capacities, individual student preferences, and a justified system of student priorities. Such models should provide a basis for transparent decision-making and the automation of allocation procedures in higher education institutions.

Thus, the research topic is relevant. The growing role of student-centered learning and elective courses creates demand for allocation models that ensure fairness, transparency, and efficiency under capacity constraints.

2. Literature review and problem statement

Allocating students to free elective courses belongs to the class of combinatorial optimization problems and is a generalization of the stable marriage problem [1]. At the same time, this problem can be interpreted as a university admissions problem [1]. In this problem, the number of places is limited and the applicant selection criteria are identical for all.

An algorithm based on a student's average grade is partially implemented in the targeted placement of state-funded places in Ukraine. Similar principles are also used in the National Resident Matching Program in the United States. In both cases, the Deferred Acceptance Algorithm (DAA) [2] is used, which was first formally studied in [1]. The idea of this algorithm lies in an iterative process of submitting applications and selecting the best candidates, which ensures an optimal allocation.

In the scientific literature, considerable attention is paid to modifications of the DAA for student course allocation. Paper [3] analyzes the Student Optimal Stable Mechanism and the Efficiency Adjusted Deferred Acceptance Mechanism, which improves efficiency but is less resistant to strategic behavior by students. Paper [4] considers similar approaches for course allocation at Harvard Business School. It studies deferred acceptance algorithm modifications and reveals potential manipulation during course selection.

Generalized versions of the DAA for allocating students among projects are presented in [5], and their automated application to course formation is discussed in [6]. In addition to stable matching algorithms, considerable attention is also paid to optimization approaches. Paper [7] analyzes algorithmic mechanisms based on matching, second-price, and optimization methods that ensure a balance between fairness and efficiency.

Paper [8] considers minimax-rank constraints for improving student allocation, while [9] proposes a Pareto-optimal allocation algorithm that takes into account real constraints, in particular credit limits and timetable conflicts. Paper [10] uses a nonlinear random matrix method for allocating places in educational institutions, and [11] applies the Hungarian algorithm to search for an optimal allocation. Paper [12] proposes a fuzzy multi-criteria decision-making method for considering the priorities of students and instructors. Paper [13] investigates evolutionary algorithms for improving the quality of student project allocation.

Thus, summarizing previous studies, it can be observed that modern approaches mainly attempt to balance student preferences, overall efficiency, and fairness, often relying on heuristics or soft constraints.

Most existing systems rely on single-criterion indicators. No individual criterion is sufficient to fully capture the academic validity of choices, social fairness, and the organizational needs of higher education institutions.

To address this problem, there is an objective need to employ an integral selection criterion capable of incorporating the entire set of relevant factors. Some factors naturally have a quantitative nature, including academic performance, seniority, and registration time. For non-quantitative criteria, such as course relevance, inclusiveness, and motivation, appropriate scales can convert qualitative assessments into quantitative measures.

The integral criterion makes it possible to transform a complex selection procedure into a transparent and universally understandable quantitative rule. Moreover, it ensures flexibility: through the use of weighting coefficients, universities can adapt the model to their specific priorities while preserving the general allocation principle.

Reducing student characteristics to a single numerical indicator provides a rigorous mathematical foundation for process automation. Algorithmic allocation based on optimization methods minimizes subjectivity, ensures reproducibility of results, and is particularly important for large universities.

Despite numerous approaches to allocating students to elective courses, several problems remain unresolved. Existing models do not simultaneously account for limited course capacity, individual student preferences, and student priority based on an integral rating score. They also insufficiently reflect both the order and intensity of preferences. Existing approaches primarily focus on procedural allocation mechanisms or individual selection criteria. Consequently, they fail to fully formalize a fair and transparent student allocation mechanism.

3. The aim and objectives of the study

The study aims to develop and justify mathematical models for elective course allocation under capacity constraints. These models ensure the consideration of individual preferences and rating-based student priorities. Additionally, the research identifies specific features of applying weighted and lexicographic allocation approaches.

To achieve this aim, the following objectives were defined:

– to formalize the problem of allocating students to free elective courses under conditions of limited course capacity;

– to construct mathematical allocation models for two ways of representing students' preferences, namely ranked lists and normalized weight coefficients;

– to investigate the specific features of using weighted and lexicographic approaches when taking into account students' integral rating scores;

– to determine, on the basis of the experimental study, the comparative advantages of the proposed models relative to the baseline FIFO approach.

4. Materials and methods for modeling student allocation to elective courses

4.1. Research hypothesis, object, and subject of the study

The hypothesis of the study is that optimization models based on an integral rating score and student preferences make it possible to obtain a more justified allocation of students than the FIFO approach. These models rely on both quantitative scoring and individual preferences.

The object of the study is the process of allocating students to elective courses in higher education institutions.

The subject of the study is mathematical models, optimization criteria, and algorithms for allocating students under capacity constraints.

4.2. Problem statement and input data

It is required to construct an allocation of students to elective courses that maximally satisfies their preferences, while giving priority to students with higher Integral Rating Score (IRS) values. We consider the allocation problem within a single catalogue of elective courses, since such an allocation constitutes a structural unit of the overall problem.

The input data are defined as follows:

– the number of students n;

– the number of courses in the catalogue m;

– the set of courses $D = \{D_1, D_2, \dots, D_m\}$.

For each course D_j, $j = 1, \dots, m$, the admissible bounds on the number of enrolled students are specified:

– the minimum number of students allowed to enroll in course D_j, $q_j^{\min}$;

– the maximum number of students allowed to enroll in course D_j, $q_j^{\max}$;

– the number of elective courses in an individual student's study plan for the considered semester, v; each student is assigned to exactly v courses from the set D.

For the correctness of the problem, it is assumed that

$$\sum_{j=1}^{m} q_j^{\min} \le nv \le \sum_{j=1}^{m} q_j^{\max}, \tag{1}$$

that is, the total capacity of the courses is sufficient to ensure the existence of a feasible allocation.

Each student i, $i = 1, \ldots, n$, is characterized by an integral rating score g_i. The integral rating score is used as a generalized quantitative indicator of student priority in the allocation process. It aggregates several criteria that may be relevant for determining the order of access to limited elective-course capacity. In general form, the integral rating score of student can be calculated as a weighted sum of normalized partial indicators.

Student i's preferences regarding the selection of courses can be specified in one of two ways.

Option 1 (ranked list). Student i provides an ordered list of courses $\left(D^i_{j_1}, D^i_{j_2}, \ldots, D^i_{j_m}\right)$, sorted in descending order of priority, which is a permutation of the set $\{D_1, D_2, \ldots, D_m\}$. In this case, course $D^i_{j_1}$ has the highest priority for student i, whereas $D^i_{j_m}$ has the lowest.

Option 2 (weight coefficients). Student i specifies weight coefficients w_{ij} for each course D_j, $j = 1, \ldots, m$, satisfying the normalization condition

$$\sum_{j=1}^{m} w_{ij} = 1, \quad w_{ij} \geq 0. \tag{2}$$

Allocation scenarios.

Depending on the relationship between the total maximum capacity and the required number of student placements, two scenarios are possible.

Scenario 1. If

$$\sum_{j=1}^{m} q_j^{\max} > nv, \tag{3}$$

then the allocation process is carried out in two stages: a preliminary stage and a final stage.

Scenario 2. If

$$\sum_{j=1}^{m} q_j^{\max} = nv, \tag{4}$$

then the allocation is performed in a single (final) stage.

Allocation stages.

Stage 1 (preliminary). At this stage, the number of available positions q_j for each course D_j is determined such that the following constraints are satisfied:

$$q_j^{\min} \leq q_j \leq q_j^{\max}, \quad j = 1, \ldots, m, \tag{5}$$

and

$$\sum_{j=1}^{m} q_j = nv. \tag{6}$$

During the preliminary stage, students' preferences are analyzed. As a result, courses that fail to attract the minimum required number of students $q_j^{\min}$ may be excluded from further consideration.

Stage 2 (final). At this stage, the final allocation of students to courses is performed in accordance with the values q_j determined at the preliminary stage. The allocation should ensure the most comprehensive satisfaction of student preferences, taking into account their IRS scores.

The scheme of allocating students to elective courses obtained as a result of Stage 2 is presented in Fig. 1.

D_1	i_1^1	i_2^1	...	...	$i_{q_1}^1$			
...								
D_j	i_1^j	i_2^j			...			$i_{q_j}^j$
...								
D_m	i_1^m	i_2^m		...		$i_{q_m}^m$		

Fig. 1. Scheme of allocation of students to elective courses

4.3. Mathematical models of student allocation

At the first stage, the values q_j, $j = 1, \ldots, m$, are determined. These values define the number of available positions in courses D_j and satisfy the following constraints

$$q_j^{\min} \leq q_j \leq q_j^{\max}, \quad j = 1, \ldots, m, \tag{7}$$

$$\sum_{j=1}^{m} q_j = nv. \tag{8}$$

In essence, this stage is implemented as a simple feasibility-checking procedure that does not require the use of a complex mathematical framework or optimization methods. For each course, the possibility of assigning an admissible number of places is sequentially verified, taking into account both lower and upper capacity bounds, and student preferences are also analyzed. As a result of this analysis, courses for which it is impossible to ensure the minimum required number of students $q_j^{\min}$ may be excluded from further consideration. Consequently, a consistent capacity vector $(q_1, \ldots, q_m)$ is formed, which is then used at the second stage of the allocation process.

Mathematical model based on a weighted approach.

At the second stage, the final allocation of students to courses must be performed, assuming that the number of available positions q_j, $j = 1, \ldots, m$, has already been determined for each course.

The decision variables are binary

$$x_{ij} = \begin{cases} 1, & \text{if student } i \text{ is assigned to course } D_j, \\ 0, & \text{otherwise,} \end{cases} \tag{9}$$

where $i = 1, \ldots, n$, $j = 1, \ldots, m$.

For each course D_j, the number of assigned students must be equal to the predefined capacity

$$\sum_{i=1}^{n} x_{ij} = q_j, \quad j = 1, \ldots, m. \tag{10}$$

Each student must be assigned to exactly v courses

$$\sum_{j=1}^{m} x_{ij} = v, \quad i = 1, \ldots, n. \tag{11}$$

Additionally,

$$x_{ij} \in \{0, 1\}, \quad i = 1, \ldots, n, \quad j = 1, \ldots, m. \tag{12}$$

Objective function. The objective is to construct an allocation that maximally satisfies student preferences while simultaneously giving preference to students with higher rating scores.

The formalization of the objective function depends on how student preferences are specified.

Option 1. Preferences given as a ranked list.

Let r_{ij} denote the rank of course D_j in the preference list of student i : $r_ij = 1$ corresponds to the highest priority, and $r_{ij} = m$ to the lowest.

In this case, the higher the priority of a course for a student, the greater its contribution to the objective function. Taking into account the student's rating score, the objective function can be written as

$$F_1(x) = \sum_{i=1}^{n} \sum_{j=1}^{m} g_i(m - r_{ij} + 1)x_{ij} \rightarrow \max . \tag{13}$$

Thus, the model seeks an allocation that simultaneously ensures high overall student satisfaction and prioritization of students with higher ratings.

Option 2. Preferences given as weight coefficients.

In this case, the objective function is naturally defined as

$$F_2(x) = \sum_{i=1}^{n} \sum_{j=1}^{m} g_i w_{ij} x_{ij} \rightarrow \max . \tag{14}$$

The proposed models employ a weighted approach in which all students are considered simultaneously, and each student's contribution to the objective function is scaled by their rating score. This approach is straightforward to implement and transparent in interpretation.

At the same time, a situation may arise where a partial deterioration in the outcome for a high-ranked student is compensated by a significant improvement for several other students. If the aggregate effect is larger, such a solution may be considered optimal by the model. Therefore, incorporating rating scores into the objective function ensures a weighted priority but does not guarantee a strict priority.

Mathematical model based on the lexicographic approach.

If it is required to enforce a rule under which students with higher rating scores must have unconditional priority over those with lower scores, a simple weighted objective function is insufficient. In this case, a lexicographic approach should be applied. Its essence lies in sequential optimization. First, allocation quality is maximized for highest-rated students. Then, among these optimal solutions, quality is maximized for the next group.

Formation of priority groups of students. Let each student i be characterized by a rating score g_i. To implement prioritization based on these scores, the set of all students is partitioned into disjoint groups

$$G_1, G_2, \ldots, G_K, \tag{15}$$

where G_1 is the group of students with the highest rating scores, G_2 is the group with the next level of ratings, and so forth. Thus, the groups are ordered in descending priority.

Such a partition enables a strict priority hierarchy. During optimization, preferences of group G_1 are maximally satisfied first. Then, group G_2 is optimized while preserving optimality for G_1.

For practical purposes, it is reasonable to form groups based not on exact rating values but on intervals of IRS.

The decision variables and constraints remain the same as in the previous model.

Objective function for ranked preferences.

For each priority group G_k, $k = 1, \ldots, K$, define a partial objective function

$$F_1^k(x) = \sum_{i \in G_k} \sum_{j=1}^{m} (m - r_{ij} + 1)x_{ij} . \tag{16}$$

In this case, the problem consists in the lexicographic maximization of the vector of objective functions

$$(F_1^1(x), F_1^2(x), \dots, F_1^K(x)) \to \text{lex-max}. \quad (17)$$

This means that first the value $F_1^1(x)$ is maximized, i.e., the total satisfaction of students in the highest-priority group. Then, among all allocations optimal with respect to $F_1^1(x)$, the value $F_1^2(x)$ is maximized, and similarly for subsequent groups.

Objective function for weighted preferences.

For each group G_k, the partial objective function is defined as

$$F_2^k(x) = \sum_{i \in G_k} \sum_{j=1}^{m} w_{ij} x_{ij}, \quad k = 1, \dots, K. \quad (18)$$

In this case, the problem also consists in the lexicographic maximization of the vector

$$(F_2^1(x), F_2^2(x), \dots, F_2^K(x)) \to \text{lex-max}. \quad (19)$$

Thus, depending on the method of representing student preferences and the approach to incorporating rating scores, four mathematical models of allocation can be distinguished:

– model M1: ranked preferences with a weighted approach;
– model M2: ranked preferences with a lexicographic approach;
– model M3: weighted preferences with a weighted approach;
– model M4: weighted preferences with a lexicographic approach.

4.4. Solution methods and algorithmic implementation

The formulated problems of assigning students to elective courses belong to the class of discrete optimization problems, since the decision variables are binary and represent whether a student is assigned to a particular course.

Depending on the representation of student preferences and the approach to incorporating rating scores, different solution methods can be applied.

For models based on the weighted approach, the problem reduces to a binary Integer Linear Programming (ILP) problem. The objective function is linear, and the constraints enforce requirements on the number of students assigned to each course and the number of courses assigned to each student. Exact methods of integer optimization can be applied, including the branch-and-bound method, cutting-plane methods, and their combinations. The main advantage of this approach is the ability to obtain an optimal solution within the given mathematical model.

For models based on the lexicographic approach, the problem also belongs to binary ILP, but it is solved sequentially. First, the objective function is maximized for the highest-priority group of students. Then, the obtained optimal value is fixed as an additional constraint, and the problem is solved for the next group. This procedure is repeated for all priority groups. Such an approach ensures strict prioritization according to rating scores but is computationally more demanding.

For large-scale problems or under time constraints, heuristic and metaheuristic methods may also be applied. These include greedy algorithms, local search, genetic algorithms, simulated annealing, and other approximate procedures. While they do not guarantee a globally optimal solution, they allow obtaining feasible and sufficiently high-quality allocations within reasonable computational time.

In this study, preference is given to exact methods, as they ensure the determination of an optimal solution, which is essential for the correct analysis and comparison of the developed models. Their application is also justified by the size of practical problems and the requirement to obtain a well-founded allocation that strictly satisfies all imposed constraints.

It should be noted that for weighted models (M1 and M3), the solution process is single-stage. It reduces to the direct application of binary ILP methods to maximize the global objective function

$F_1(x)$ or $F_2(x)$ under the given constraints. In contrast, the lexicographic approach (models M2 and M4) requires an iterative solution procedure.

Pseudocode of the generalized lexicographic approach algorithm.

```
1. Input:
        I = {1, 2, ..., n} - set of students
        J = {1, 2, ..., m} - set of courses
        g_i - integral rating score of student i (i ∈ I)
        q_j - number of available positions in course D_j (j ∈ J)
        v - number of courses each student must be assigned to
        b - type of preference representation (b = 0 for ranked lists, b = 1 for weight coefficients)
        rankPrefs - student preferences in the form of ranked lists
        weightPrefs - student preferences in the form of weight coefficients
2. Output:
        X = (x_ij) - matrix of the final allocation of students to courses
3. Partition the set of students I into priority groups G_1, G_2, ..., G_K
4. for k = 1, 2, ..., K do:
5.      if b = 0 then
6.          Solve optimization problem F_1^k(x) → max using rankPrefs under the common set of constraints and all additional constraints obtained in previous steps
7.          Add the constraint F_1^k(x) = z_k^*, where z_k^* is the optimal value of F_1^k(x)
8.      else
9.          Solve optimization problem F_2^k(x) → max using weightPrefs under the common set of constraints and all additional constraints obtained in previous steps
10.         Add the constraint F_2^k(x) = z_k^*, where z_k^* is the optimal value of F_2^k(x)
11.     end if
12. end for
13. return X
```

The computational complexity of models M1 and M3 is determined by solving a single ILP problem with $n \cdot m$ binary variables and $n+m$ linear constraints. In the general case, this problem belongs to the class of NP-hard problems; therefore, its theoretical worst-case complexity may grow exponentially with an increasing number of binary variables.

The complexity of the lexicographic models M2 and M4 is higher because they require the sequential solution of K binary ILP problems, where K is the number of priority groups. Therefore, compared with the weighted models, the lexicographic approach preserves the NP-hardness of the underlying optimization problem. At the same time, it increases computational costs due to the need for repeated sequential solution procedures for all priority groups.

5. Results of investigating mathematical models for student allocation to elective courses

5.1. Experimental design

The experimental study was organized as two separate experiments differing in the method of representing student preferences. In the first experiment, preferences were specified as ranked lists, while in the second experiment they were given as normalized weight coefficients for each course. Accordingly, in the first experiment, allocation strategies were compared using the objective function $F_1(x)$, and in the second using $F_2(x)$. In each experiment, three allocation strategies were considered.

These include FIFO, where priority is given to students who submitted their requests earlier, the weighted approach implemented by models M1 and M3, and the lexicographic approach implemented by models M2 and M4.

To evaluate the effectiveness of the proposed models, a simulation of the allocation process was conducted. A test dataset was generated with parameters corresponding to a realistic faculty scale: $n = 1000$ students and $m = 30$ elective courses. Each student was required to be assigned to exactly $v = 3$ courses.

To approximate real educational conditions, non-uniform demand for courses was introduced: a subset of highly popular ("high-demand") courses was defined, creating an artificial shortage of available positions and increased competition. Students were sorted by rating scores and divided into four equal-sized priority groups.

To ensure statistical reliability and reduce the influence of stochastic factors (such as random registration order and preference generation), the experiments were conducted as a series of 100 Monte Carlo simulations.

To evaluate the computational feasibility of the proposed models, the empirical complexity of the considered problem instances was also assessed. For the experimental parameters $n = 1000$ and $m = 30$, the number of binary variables is $n \cdot m = 30000$, while the number of main linear constraints is $n + m = 1030$. Such a problem size is acceptable for modern integer programming solvers. For weighted models M1 and M3, expected solution times range from seconds to minutes. This duration depends on hardware, solver settings, and generated data structure. For the lexicographic models M2 and M4, the execution time may be up to K times higher, since the problem is solved sequentially for each priority group.

5.2. Results for rank-based preference representation

In the first experiment, student preferences were represented as ranked lists of elective courses. The averaged results for the FIFO strategy, the weighted model M1, and the lexicographic model M2 are shown in Figs. 2–4.

5.2.1. Results by the average rank criterion

The average rank of assigned courses for each priority group is shown in Fig. 2, where a lower value corresponds to a better allocation result.

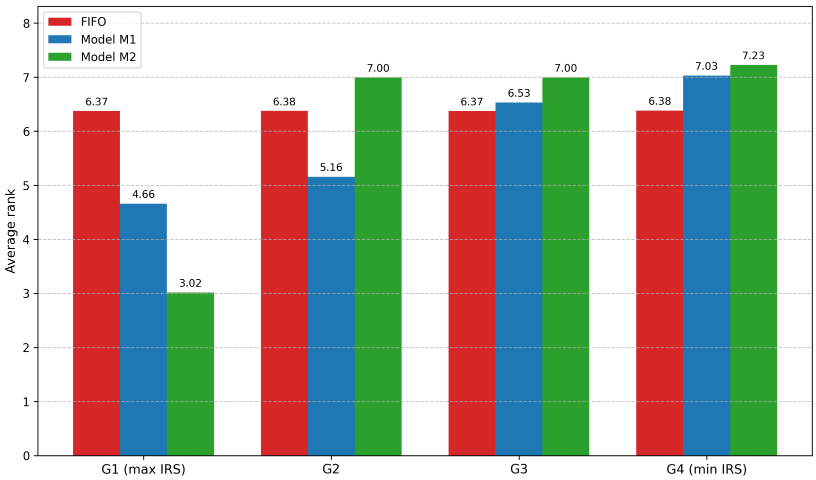

Fig. 2. Average rank of assigned courses (Experiment 1)

Under the FIFO strategy, the average rank is almost identical for all student groups and ranges from 6.37 to 6.38. For the weighted model M1, the average rank changes from 4.66 for the highest-priority group to 7.03 for the lowest-priority group. For the lexicographic model M2, the highest-priority group has an average rank of 3.02, whereas the corresponding values for the remaining groups lie within the range from 7.00 to 7.23.

5.2.2. Results by the preference satisfaction criterion

The level of student preference satisfaction in relative terms is presented in Fig. 3, where 100% corresponds to the ideal allocation, that is, receiving courses ranked 1, 2, and 3.

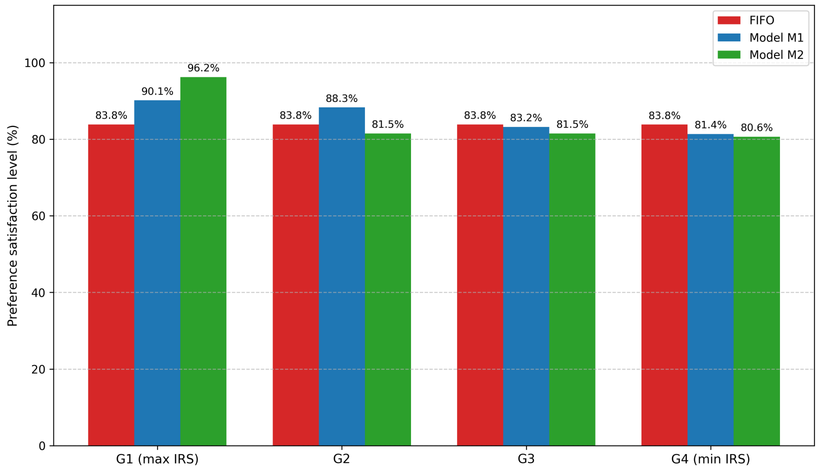

Fig. 3. Satisfaction level of student preferences (Experiment 1)

Under FIFO, the satisfaction level is about 83.8% for all groups. In model M1, the satisfaction level of the top group reaches 90.1%, while the values for the lower-priority groups gradually decrease. In model M2, the satisfaction level of the highest-priority group reaches 96.2%, whereas for the remaining groups it is about 81%.

5.2.3. Results by the distribution of desirable courses

The distribution of the number of assigned desirable courses is shown in Fig. 4, where desirable courses are defined as those belonging to the student's individual Top-3 list.

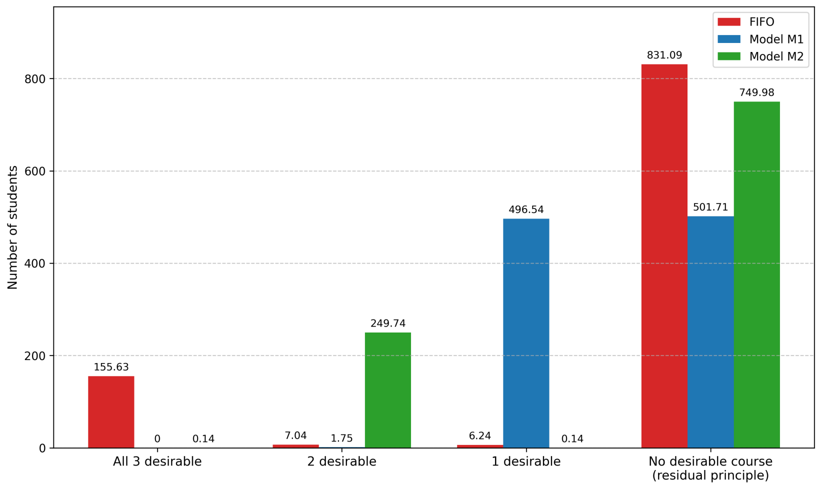

Fig. 4. Efficiency of allocating desired courses (Experiment 1)

Under FIFO, about 155 students obtain all three desirable courses, while about 831 students do not obtain any of them. In model M1, approximately 496 students obtain at least one desirable course. In model M2, about 250 students obtain exactly two desirable courses.

5.3. Results for weight-based preference representation

In the second experiment, student preferences were represented by normalized weight coefficients assigned to each course. The averaged results for the FIFO strategy, the weighted model M3, and the lexicographic model M4 are shown in Figs. 5–7.

5.3.1. Results by the average rank criterion

The average rank of assigned courses for the second experiment is presented in Fig. 5.

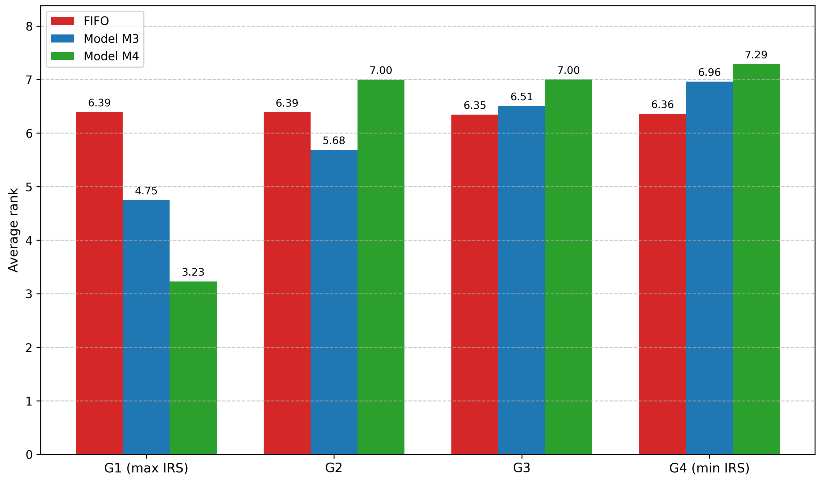

Fig. 5. Average rank of assigned courses (Experiment 2)

Under FIFO, the average rank remains within the range from 6.35 to 6.39 for all groups. In the weighted model M3, this indicator ranges from 4.75 for the highest-priority group to 6.96 for the lowest-priority group. In the lexicographic model M4, the highest-priority group has an average rank of 3.23, while the remaining groups have values from 7.00 to 7.29.

5.3.2. Results by the preference satisfaction criterion

The satisfaction level obtained in the second experiment is shown in Fig. 6.

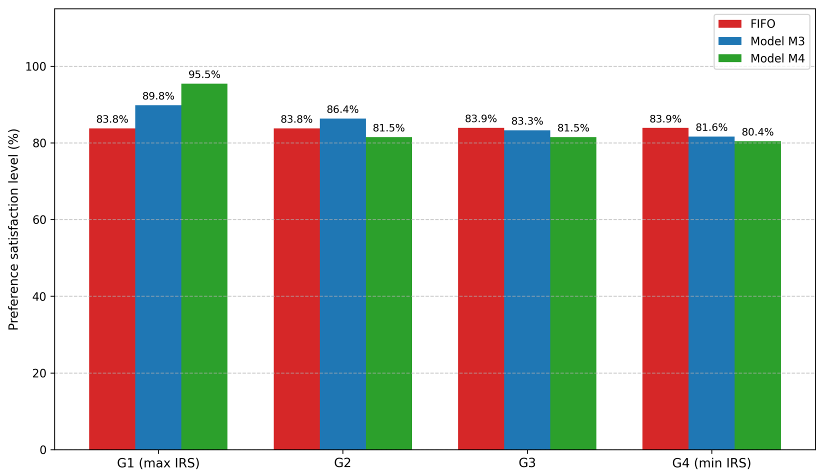

Fig. 6. Satisfaction level of student preferences (Experiment 2)

In model M3, the satisfaction level of the strongest students reaches 89.8%. In model M4, the satisfaction level of the highest-priority group reaches 95.5%.

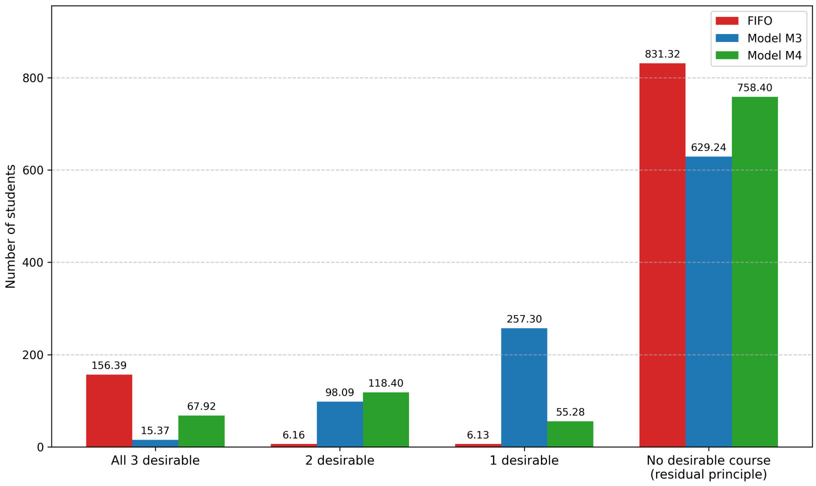

Fig. 7. Efficiency of allocating desired courses (Experiment 2)

5.3.3. Results by the distribution of desirable courses

The distribution of the number of assigned desirable courses in the second experiment is presented in Fig. 7.

In model M3, about 629 students remain outside their Top-3 list, while the share of students obtaining one or two desirable courses increases. In model M4, 67 students obtain all three desirable courses, 118 obtain two desirable courses, and 55 obtain one desirable course.

6. Discussion of the results of student allocation modeling for elective courses

6.1. Interpretation of the obtained results

The obtained results show that the FIFO strategy does not ensure any meaningful differentiation between student groups according to the integral rating score. In both experiments, the indicators for all groups remain nearly identical, which means that the order of registration acts as the dominant factor and effectively produces a quasi-random access mechanism to popular elective courses.

At the same time, the weighted models M1 and M3 demonstrate a gradual deterioration of indicators from the highest-priority group to the lowest-priority group. This pattern indicates that these models do account for student priority according to the integral rating score, but do so in a soft manner. The optimization process in these models seeks to maximize the global objective function, which makes it possible to improve the overall allocation quality without introducing an excessively sharp decline in outcomes for lower-priority groups. As a result, the weighted approach produces a more balanced distribution between efficiency and fairness.

The lexicographic models M2 and M4 produce a fundamentally different pattern. Their results demonstrate a clear concentration of advantages in the highest-priority group, while the indicators of the other groups become substantially worse. This is consistent with the nature of lexicographic optimization. In this approach, the satisfaction of higher-priority groups is maximized first, while lower-priority groups are considered only after fixing the best achievable result for the preceding ones. Thus, the lexicographic approach ensures a strict priority hierarchy.

A comparison of the two experiments also reveals an important difference between rank-based and weight-based preference modeling. In the rank-based setting, the models account only for the order of student preferences. In the weight-based setting, they additionally reflect the intensity of those preferences. This makes the distribution more flexible, since the model can distinguish between courses that are merely preferred and courses that are critically important for a particular student. Such flexibility is especially visible in model M4, where the distribution of desirable courses within the top group is more differentiated than in model M2.

6.2. Comparison of the proposed models and practical implications

The results suggest that the choice of the allocation model should depend on the policy priorities of the higher education institution. If the main objective is to maximize total satisfaction and to distribute scarce places in a more balanced way, then the weighted models M1 and M3 are more appropriate. They reduce extreme outcomes and increase the share of students who receive at least one desirable course.

If, however, the institution considers the strict observance of rating-based hierarchy to be the principal requirement, then the lexicographic models M2 and M4 are more appropriate. These models provide an explicit and mathematically well-defined mechanism for giving unconditional preference to students with higher integral rating scores. This may be relevant in educational systems where academic performance must be reflected directly in access to high-demand elective courses.

The comparison between the rank-based and weight-based versions of the models further indicates that the use of weight coefficients provides additional expressive power. Unlike ranked lists, weight coefficients allow the model to capture not only the order of student interests but also their strength. Therefore, models M3 and M4 can be considered more flexible instruments for representing individual educational demand.

From a practical perspective, the proposed models can serve as a basis for the development of transparent and reproducible decision-support mechanisms in universities. Their implementation would make it possible to formalize the allocation process, reduce the influence of purely technical factors such as registration timing. It would also allow adapting allocation rules to the strategic priorities of a specific institution or educational program.

6.3. Limitations of the study and prospects for further research

The conducted study has several limitations. First, the experiments were carried out on simulated data, although their size and structure were chosen to approximate real university conditions. Second, the problem was considered within a single catalog of elective disciplines, which simplifies the actual organization of student choice. Third, the current models do not account for timetable conflicts, inter-catalog dependencies, or multi-semester planning constraints.

Further research may therefore be directed toward validating the proposed models on real institutional data, extending them to multiple catalogs of elective courses, and incorporating scheduling constraints. It may also focus on developing hybrid exact-heuristic solution procedures for larger-scale allocation problems. Another promising direction is the refinement of the integral rating score through the inclusion of additional academic, social, and organizational criteria that reflect the specific priorities of a higher education institution.

Conclusion

The paper solves the scientific and practical problem of developing mathematical models for allocating students to free elective courses under conditions of limited capacity. These models take into account students' individual preferences and their priority based on an integral rating score.

As a result of the study, the problem of allocating students to free elective courses is formalized as a discrete optimization problem. This formulation takes into account course capacity constraints, the number of courses selected by each student, and feasibility conditions for the allocation.

Four mathematical allocation models are developed, differing in the way students' preferences are represented and in the mechanism used to incorporate student priority. Unlike models that rely solely on ranking, models based on normalized weight coefficients make it possible to account not only for the order but also for the intensity of students' individual preferences.

The experimental study shows that the baseline FIFO approach does not in fact ensure differentiation of allocation outcomes according to students' integral rating scores. It is shown that the weighted models provide a more balanced allocation and a higher overall level of student satisfaction, whereas the lexicographic models ensure stricter adherence to the hierarchy of priorities.

It is also established that the use of weighted preference representation expands the possibilities for modeling individual educational demand.

The obtained results make it possible to formulate the scientific novelty of the study as follows:

1. for the first time, a comprehensive set of mathematical models for allocating students to free elective courses has been developed. These models combine two ways of representing students' preferences – ranked lists and normalized weight coefficients – with two approaches to incorporating student priority based on an integral rating score: weighted and lexicographic;
2. the approach to formalizing students' preferences in the allocation problem has been improved through the use of normalized weight coefficients. This makes it possible to account not only for the order but also for the intensity of individual preferences;
3. the application of lexicographic optimization to the problem of allocating students to elective courses has been further developed. This ensures stricter adherence to the hierarchy of priorities compared with procedural mechanisms of the FIFO type.

Thus, the proposed models create a theoretical foundation for the development of transparent and formalized mechanisms for allocating students to free elective courses. The choice of a particular model should be determined by the priorities of the educational policy of a higher education institution.

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МОДЕЛІ РОЗПОДІЛУ СТУДЕНТІВ НА ДИСЦИПЛІНИ ВІЛЬНОГО ВИБОРУ

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Статтю присвячено побудові математичних моделей розподілу студентів на дисципліни вільного вибору в закладах вищої освіти за умов обмеженої кількості місць. Актуальність дослідження зумовлена потребою в прозорих і формалізованих механізмах розподілу, які, на відміну від процедур ранньої реєстрації, дають змогу враховувати не лише черговість подання заяв, а й індивідуальні уподобання здобувачів освіти та їхню пріоритетність за інтегральним рейтинговим балом. Метою дослідження є побудова та теоретичне обґрунтування математичних моделей розподілу студентів на дисципліни вільного вибору, а також виявлення особливостей застосування зваженого та лексикографічного підходів до такого розподілу. У роботі використано методи дискретної оптимізації, зокрема бінарне лінійне цілочислове програмування та лексикографічну оптимізацію. Формалізовано задачу розподілу з урахуванням обмежень на місткість дисциплін, кількість дисциплін у виборі студента та умов допустимості розподілу. Побудовано чотири математичні моделі, що поєднують ранжовані списки та нормовані вагові коефіцієнти з двома способами врахування пріоритетності студентів – зваженим і лексикографічним. Експериментальну перевірку проведено на згенерованих даних, наближених до умов великого факультету, для 1000 студентів і 30 дисциплін із використанням моделювання методом Монте-Карло. Установлено, що зважені моделі забезпечують більш збалансований розподіл і вищий рівень задоволеності студентів, тоді як лексикографічні моделі гарантують суворіше дотримання ієрархії пріоритетів. Наукова новизна полягає у побудові комплексу моделей, що дають змогу враховувати не лише порядок, а й інтенсивність індивідуальних переваг студентів. Практичне значення одержаних результатів полягає у можливості використання запропонованих моделей як основи для створення прозорих механізмів підтримки прийняття рішень у закладах вищої освіти.

Ключові слова: дисципліни вільного вибору, розподіл студентів, цілочислове програмування, лексикографічна оптимізація, інтегральний рейтинговий бал.